EXTENDED FAULT-DIAMETER OF STAR GRAPHS

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Abstract. An important issue in computer communication networks is fault-tolerant routing. A popular graph topology for interconnection network is the star graph [1]. We know from [6] that if up to $2n-4$ vertices are deleted, the resulting graph has a single large component and at most one other component of size at most two. This paper will discuss routing in the large component and also show that its diameter in the faulty star graph is bounded by $diam(S_n) + 9$.

Keywords: Interconnection networks, star graphs, extended fault-tolerant routing

1. Introduction

Distributed processor architectures offer the potential advantage of high speed, provided they are highly fault tolerant and reliable, and have good communication between remote processors. An important component of such a distributed system is its graph topology, which defines the interprocessor communication architecture. One such graph topology is the class of star graphs proposed by [1]. They have many advantages over the hypercubes, such as lower degree and a smaller diameter. Some recent papers on star graphs include [2–25].

In many interconnection communication networks, the study of fault-tolerant routing plays a major role in the analysis of the networks. The fault-tolerant routing problem can be described as follows: Given a set of faulty vertices which are deleted from the graph, find a shortest routing between two vertices efficiently. Normally the maximum number of faulty vertices allowed is the connectivity of the graph, as one requires the network to be connected. However, bounding the cardinality of the set of faults by the connectivity provides a worst-case measure as the network may not be disconnected even if more vertices have been deleted. So it is of interest to study fault-tolerant routing in this setting. There are a number of approaches to study this problem. For example, one approach is to assume the faults appear in clusters and study the so-called cluster fault-tolerant routing; we refer the reader to [16]. The connectivity of S_n is $n-1$. Under the assumption of clusters, [15–18] show that S_n can tolerate more than $n-1$ faults and study various related routing problems.

A result from [6] states that if up to $2n-4$ vertices are deleted from S_n, then the resulting graph has at most two components with one of them of size at most two. In this paper, we study the fault-tolerant routing problem with up to $2n-4$ faults, which is almost double the connectivity of S_n, without extra assumptions. This will establish the result given in [6] as a corollary.

2. Preliminaries

We assume the reader is familiar with basic terminology in graph theory, specifically girth, diameter, connectivity, optimal routing and paths. Given a graph H and a set of faults $\mathcal{F} \subseteq V(H)$ such that $H \setminus \mathcal{F}$ is connected, a routing between two vertices in $H \setminus \mathcal{F}$ is called a *fault-tolerant routing* in H.

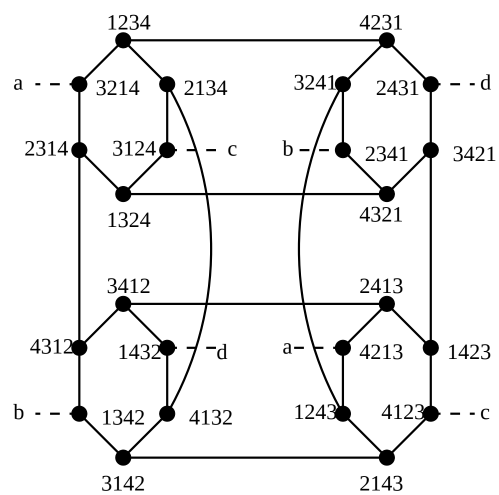

Figure 1. The star graph S_4

The star graph S_n, $(n \geq 3)$ introduced in [1], is a graph with vertex-set being the set of permutations on n symbols. Two permutations $[a_1, a_2, \ldots, a_n]$ and $[b_1, b_2, \ldots, b_n]$ are adjacent if and only if there exists an $i \neq 1$ such that $a_1 = b_i$, $a_i = b_1$ and $a_j = b_j$ for $j \notin \{1, i\}$. In other words, given two permutations, π_a and π_b, they are adjacent if one can be obtained from the other by exchanging the symbols in position 1 and position i for some $i \neq 1$. Since this change is through position i, we refer to such an edge as an *i-edge*. Figure 1 gives S_4. For example, 2413 and 1423 are adjacent through a 3-edge in S_4. It is also easy to see that S_n has girth 6. Note that if one fixes the i^{th} symbol ($i \in \{1, 2, \ldots, n\}$) in the j^{th} position ($j \in \{2, 3, \ldots, n\}$) the induced subgraph is isomorphic to S_{n-1}. We refer to this subgraph as a substar. It is easy to see that S_n is $(n-1)$-regular. It is well-known that it is $(n-1)$-connected with diameter $\lfloor \frac{3(n-1)}{2} \rfloor$ and the greedy algorithm is optimal. For example to route from $[2, 3, 1, 5, 6, 4, 7]$ to $[1, 2, 3, 4, 5, 6, 7]$, we have $[2, 3, 1, 5, 6, 4, 7]$ to $[3, 2, 1, 5, 6, 4, 7]$ to $[1, 2, 3, 5, 6, 4, 7]$ to $[5, 2, 3, 1, 6, 4, 7]$ to $[6, 2, 3, 1, 5, 4, 7]$ to $[4, 2, 3, 1, 5, 6, 7]$ to $[1, 2, 3, 4, 5, 6, 7]$. (See [1].) An important and easy consequence of the greedy routing algorithm is the following result.

Proposition 2.1. *Suppose $a = [a_1, a_2, \ldots, a_n]$ and $b = [b_1, b_2, \ldots, b_n]$ are in S_n with $n \geq 3$ and $a_i = b_i$ for some $2 \leq i \leq n$. Then there is an optimal routing from a to b such that at every intermediate vertex $c = [c_1, c_2, \ldots, c_n]$, $c_i = a_i$.*

In this paper we make use of the following theorem, and a corresponding routing algorithm, from [23]:

Theorem 2.2. *Given a set of faults $\mathcal{F}$ with $|\mathcal{F}| \leq n-2$, $S_n \backslash \mathcal{F}$ is connected and $\mathrm{diam}(S_n \backslash \mathcal{F}) \leq \mathrm{diam}(S_n)+2$. If $n \geq 7$ then $\mathrm{diam}(S_n \backslash \mathcal{F}) \leq \mathrm{diam}(S_n)+1$.*

3. Extended fault-tolerant routing in the star graphs

Throughout this section, our underlying graph is S_n. Let $\mathcal{F}$ be a set of faults with $\mathcal{F} \subseteq V(S_n)$ and $|\mathcal{F}| \leq 2n-4$. Because the size of $\mathcal{F}$ is larger than the connectivity of the graph, it is possible for all of the neighbors of a vertex to be faults, in which case the vertex is isolated. It is also possible for the neighbors of a pair of vertices connected by a single edge to all be faults, in which case both vertices will be connected only to each other. These two types of components will be referred to as *extreme* and vertices in extreme components will be called *bad*.

Given a and b in $S_n \setminus \mathcal{F}$, our objective is to find a', close to a, and b', close to b, such that the greedy routing from a' to b' in S_n avoids elements of $\mathcal{F}$. Define $N_{\mathcal{F},k}$ with $k \geq 2$ as follows: $i \in \{1, 2, \ldots, n\}$ is in $N_{\mathcal{F},k}$ if and only if there is no vertex in $\mathcal{F}$ with i in the k^{th} position. That is, $N_{\mathcal{F},k}$ consists of symbols not in the k^{th} position of any fault in S_n.

For example, in S_4, if $\mathcal{F} = \{(1, 3, 2, 4), (4, 2, 1, 3), (4, 3, 1, 2), (3, 2, 4, 1)\}$, then we have the following: $N_{\mathcal{F},2} = \{1, 4\}$, $N_{\mathcal{F},3} = \{3\}$, and $N_{\mathcal{F},4} = \emptyset$.

Lemma 3.1. *Suppose $N_{\mathcal{F},p} \neq \emptyset$ for some $p \in \{2, 3, \ldots, n\}$ and $n \geq 6$. Let $t \in N_{\mathcal{F},p}$. Then for every vertex a in $S_n \setminus \mathcal{F}$ that is not bad, there exists a fault-free path of length at most 5 from $a = [a_1, a_2, \ldots, a_n]$ to a' such that the p^{th} position of a' is t.*

Proof. Without loss of generality, we may assume $p = 2$. If $t = a_2$ then $a' = a$. If $t = a_1$ then $a' = [a_2, a_1, a_3, \ldots, a_n]$. For notational convenience, we assume $t = a_3$. We note that it is enough to find a fault-free path from a to a vertex with a_3 in its first position as $a_3 \in N_{\mathcal{F},2}$ and one can route from this vertex to the proper a' via a 2-edge. Hence, $[a_3, a_2, a_1, a_4, \ldots, a_n] \in \mathcal{F}$ or we are done. Moreover, at least one of the ends of each of the 3-edges in level 2 (Figure 2) must be in $\mathcal{F}$ or we are done. So we have identified at least $n - 1$ faults. Since a is not an isolated vertex, we may assume a neighbor of a via an i-edge where $i \in \{2, 4, 5, \ldots, n\}$ is not in $\mathcal{F}$. Without loss of generality, we may assume $i = 2$ or $i = 4$.

We first ssume $i = 4$. Therefore $[a_4, a_2, a_3, a_1, a_5, \ldots, a_n] \notin \mathcal{F}$ and $[a_3, a_2, a_4, a_1, a_5, \ldots, a_n] \in \mathcal{F}$ or we are done. If at least one of the dashed 3-edges in the last level of Figure 2 has neither of its ends belonging to $\mathcal{F}$, then we are done. Hence we have discovered the possible locations of $n - 3$ more faults bringing the total to $2n - 4$. So any vertex that has not implicitly appeared in Figure 2 is not a fault. Suppose at least one of the neighbors of $[a_4, a_2, a_3, a_1, a_5, \ldots, a_n]$ via a j-edge where $j \in \{2, 5, 6, \ldots, n\}$ is not in $\mathcal{F}$. Without loss of generality, we may assume $j = 2$ or $j = 5$. If $j = 2$, then the following fault-free path exists: $[a_2, a_4, a_3, a_1, a_5, \ldots, a_n], [a_1, a_4, a_3, a_2, a_5, \ldots, a_n], [a_3, a_4, a_1, a_2, a_5, \ldots, a_n]$. If $j = 5$, then the following fault-free path exists: $[a_5, a_2, a_3, a_1, a_4, a_6, \ldots, a_n]$, $[a_1, a_2, a_3, a_5, a_4, a_6, \ldots, a_n], [a_3, a_2, a_1, a_5, a_4, a_6, \ldots, a_n]$. So we may assume all the solid boxed vertices in Figure 2 are faults. Moreover, we know the two round boxed vertices are faults. Since a is not a vertex in a component with two vertices, at least one of the dashed boxed vertices, other than $[a_4, a_2, a_3, a_1, a_5, \ldots, a_n]$, is not a fault. Without loss of generality, we may consider two cases. In the first case, we may conclude $[a_2, a_1, a_3, a_4, \ldots, a_n] \notin \mathcal{F}$. Moreover, the following fault-free path exists: $[a_2, a_1, a_3, a_4, a_5 \ldots, a_n], [a_4, a_1, a_3, a_2, a_5 \ldots, a_n], [a_3, a_1, a_4, a_2, a_5 \ldots, a_n]$. In the second case, we have $[a_n, a_2, a_3, \ldots, a_{n-1}, a_1] \notin \mathcal{F}$; moreover, the following fault-free path exists: $[a_n, a_2, a_3, \ldots, a_{n-1}, a_1], [a_2, a_n, a_3, \ldots, a_{n-1}, a_1]$, $[a_3, a_n, a_2, \ldots, a_{n-1}, a_1]$. So if $i = 4$, that is, the neighbor of a via the 4-edge is good, then we are done.

We now assume $i = 2$. Therefore $[a_2, a_1, a_3, a_4, a_5, \ldots, a_n] \notin \mathcal{F}$ and $[a_3, a_1, a_2, a_4, a_5, \ldots, a_n] \in \mathcal{F}$ or we are done. We still have at least $n - 2$ faults from the ends of the 3-edges in level two. Similar to the $i = 4$ case, if at least one of the 3-edges extending from the neighbors of $[a_2, a_1, a_3, a_4, a_5, \ldots, a_n]$ has neither of its ends belonging to $\mathcal{F}$, then we are done. Hence we have discovered the locations of $n - 2$ more faults, bringing the total to $2n - 4$. Suppose at least one of the neighbors of $[a_2, a_1, a_3, a_4, a_5, \ldots, a_n]$ via a j-edge where $j \in \{4, 5, \ldots, n\}$ is not in $\mathcal{F}$. Without loss of generality, we may assume $j = 4$, so $[a_3, a_1, a_4, a_2, a_5, \ldots, a_n] \in$

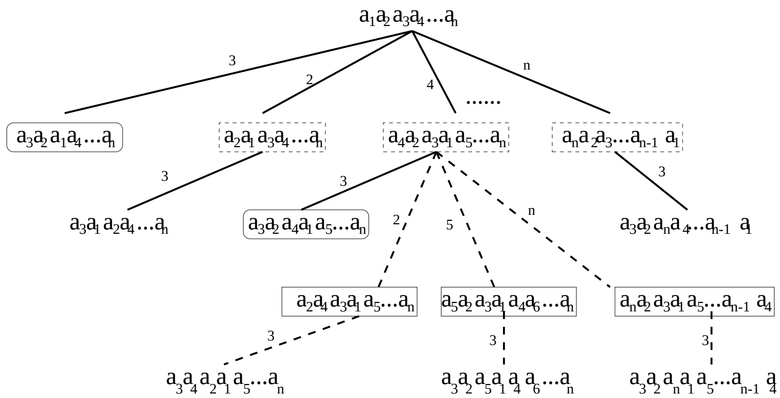

FIGURE 2. When $i = 4$ in Lemma 3.1

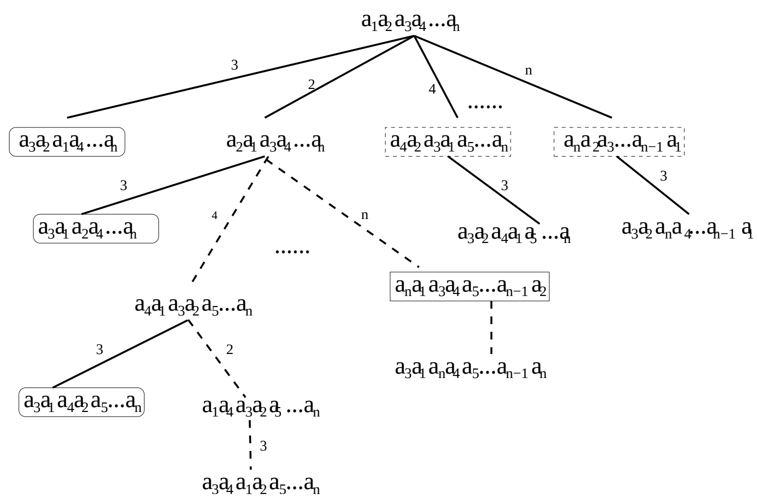

FIGURE 3. When $i = 2$ in Lemma 3.1

$\mathcal{F}$. Then the following fault-free path exists: $[a_4, a_1, a_3, a_2, a_5, \ldots, a_n]$, $[a_1, a_4, a_3, a_2, a_5, \ldots, a_n], [a_3, a_4, a_1, a_2, a_5, \ldots, a_n]$. $\square$

So, if we can find a position in which some symbol does not appear, then we can route between any two vertices that are not bad. In general, though, we cannot guarantee that this is the case. So we need to find a way to route when the faults are spread around more evenly.

Lemma 3.2. *Let $|\mathcal{F}| \leq 2n-4$, where $n \geq 6$, and suppose $N_{\mathcal{F},k} = \emptyset$ for all $k \in \{2, \ldots, n\}$, then there exists a fault-free path between any vertices $a, b \notin \mathcal{F}$ of length at most $2\mathrm{diam}(S_n) + 3$.*

Proof. Let $a = [a_1, a_2, \ldots, a_n] \notin \mathcal{F}$ and $b = [b_1, b_2, \ldots, b_n] \notin \mathcal{F}$. Note that if there exists a $j \in \{2, 3, \ldots, n\}$ such that $a_j = b_j$, then a and b are in the subgraph with a_j fixed in the j^{th} position. Then by Theorem 2.2 we can find a fault free path from a to b with length at most $diam(S_{n-1}) + 2$. So assume that $a_j \neq b_j \; \forall j \in \{2, 3, \ldots, n\}$. Then there exists a $f \in \mathcal{F}$ such that f has i in the n^{th} position, for all $i \in \{1, 2, \ldots, n\}$. Let γ_i = the number of faults with i in the n^{th} position. Note that $\gamma_i \geq 1$ as $N_{\mathcal{F},n} = \emptyset$. Now let H_i be the subgraph induced by vertices with i in the n^{th} position. Then no H_i can contain more than $n-3$ faults. So $1 \leq \gamma_i \leq n-3$. Hence $H_i \setminus \mathcal{F}$ is connected since H_i is isomorphic to S_{n-1} which is $(n-2)$-connected. Of course $a \in H_{a_n}$. We know that $b \notin H_{a_n}$. We also know that $b \in H_{b_n}$. The number of independent edges between H_{a_n} and H_{b_n} is $(n-2)! > 2n-4$ for $n \geq 6$. This means there is an edge uv between H_{a_n} and H_{b_n} where $u \in H_{a_n}$, $v \in H_{b_n}$, and $u, v \notin \mathcal{F}$. Therefore, we can route from a to u in $H_{a_n} \setminus \mathcal{F}$, from u to v, and from v to b in $H_{b_n} \setminus \mathcal{F}$ in at most $diam(S_{n-1})+2+1+diam(S_{n+1})+2 \leq 2diam(S_{n-1})+5 \leq 2diam(S_n)+3$. □

By Theorem 2.2, if $N_{\mathcal{F},p} = \emptyset$ for each $p \in \{2, 3, \ldots, n\}$, we can route between any two vertices in S_n, that is, the graph is still connected. Under this assumption, the extreme cases do not appear as the faults are evenly distributed. We can now prove Theorem 3.3 in [6]

Theorem 3.3. *If S_n is a star with at most $2n-4$ faults, $\mathcal{F}$, then $S_n \setminus \mathcal{F}$ satisfies one of the following conditions:*

1.) *$S_n \setminus \mathcal{F}$ is connected.*
2.) *$S_n \setminus \mathcal{F}$ has two components, one of which has exactly one vertex.*
3.) *$S_n \setminus \mathcal{F}$ has two components, one of which is K_2, that is, the graph with two vertices and one edge connecting them.*

Proof. If $S_n \setminus \mathcal{F}$ is connected, then we are done. If not, Lemmas 3.1 and 3.2 imply that it has exactly one big component and several small components of size one or two. Now assume there are two single-vertex components. Each vertex requires $n-1$ deleted vertices to isolate it from the rest of the graph. If neighbors of these two vertices have at most one vertex in common, then we would have $2n-3 > 2n-4$ faults. Therefore the two single, isolated vertices must have at least two common neighbors. This is impossible because S_n has girth 6. So $S_n \setminus \mathcal{F}$ has at most one singleton.

If there was a component consisting of two vertices and an edge joining them, we would have to remove $n-2$ neighbors for each vertex, or $2n-4$ distinct faults. Therefore the neighbor-set of any other small component of size one or two would be contained in this set of $2n-4$ deleted vertices. Since $n \geq 6$, this would induce a 4-cycle or 5-cycle which is impossible in S_n. The conclusion follows. □

4. Tightening the Bound

The results in the previous theorem are not as good as we could hope for. In the following theorem we will show that it is possible to tighten this bound to $diam(S_n) + c$, where c is a constant.

Theorem 4.1. *Let $\mathcal{F}$ be a set of faults in a star graph, S_n, $n \geq 6$. Then, if $|\mathcal{F}| \leq 2n-4$, there is a path between any two vertices, neither of which is bad, of length at most* $\mathrm{diam}(S_n) + c$ *where c is a constant. Moreover, when the conditions of Lemma 3.1 hold, $c \leq 9$, otherwise, $c \leq 4$.*

Proof. If the conditions of Lemma 3.1 hold (that is, the faults are clustered), then we know from that same lemma that the path is of length at most $diam(S_n) + 9$. Note that this is +9, not +10, as we are routing in S_{n-1} as one position is fixed. If the conditions of Lemma 3.1 do not hold, then there are at most $n - 3$ faults in each substar, as in Lemma 3.2. Let $a = [a_1, a_2, \ldots, a_n]$ and $b = [b_1, b_2, \ldots, b_n]$, where a is the source and b is the destination. Also, let k be the index where $b_k = a_1$.

Case 1: Assume, $k \in \{2, 3, \ldots, n\}$. Define H_j to be the substar of S_n induced by fixing j in the k^{th} position for $j \in \{1, 2, \ldots, n\}$. If the neighbor of a via the k-edge is not a fault, then a is connected to a member of the substar that contains b by a single edge and we are done by applying Theorem 2.2. So we may assume the neighbor connected to a via the k-edge is a fault.

Let σ be the permutation on $\{1, 2, \ldots, n\}$ such that $[a_1, a_2, \ldots, a_n] = [b_{\sigma(1)}, b_{\sigma(2)}, \ldots, b_{\sigma(n)}]$. For each $i \in \{2, 3, \ldots, k-1, k+1, \ldots, n\}$, consider the path of length two from a via the i-edge followed by the $\sigma(i)$-edge. Note that none of the edges are k-edges along these two-paths, so all the vertices in these paths have a_k in the k^{th} position. Hence all of the vertices are in the same substar, *i.e.* H_{a_k}, which has at most $n - 3$ faults. Because there are $n-2$ such two-paths, at least one of them has no faults. Note that these paths are vertex disjoint as S_n has girth 6. Suppose the two-path leaving a on an l-edge has no faults, where $l \neq k$. The end of this two-path is not a fault and matches b in the $\sigma(l)^{th}$ position. Now consider the substar induced by fixing a_l in the $\sigma(l)^{th}$ position. This substar contains both b and the end of the two-path without faults. By Theorem 2.2 we may route between these two vertices, and in fact, the length of such a routing is at most $diam(S_{n-1}) + 2 \leq diam(S_n) + 1$. Therefore we can route from a to b in at most $diam(S_n) + 3$ steps.

Case 2: Assume $k = 1$. Then, because a is not an isolated vertex, it has a neighbor. This neighbor will not match b in the first position, so we can route from this neighbor to b using the same procedure as in the first case with at most one extra edge, and so the path has length at most $diam(S_n) + 4$.

□

5. Concluding Remarks

In this paper, we show that simple routing still exists in a star graph when the number of faults is larger than the connectivity of the graph, but still "small" enough. In the first instance, there is a substar that contains no faults, and so we can route into this star and not worry about the faults. This corresponds to faults appearing in clusters, or unevenly spread, and so some parts of the graph are relatively untouched. The second instance occurs when the faults are distributed more evenly. This turns out to be the better case, in that the worst case routing does not need too many extra steps. Intuitively this makes sense because if the faults are not evenly distributed a short path will not be diverted too far. It might be interesting to investigate whether the bound in the first instance can be tightened further, but it is probably not worthwhile. For example, if $n \geq 7$, we can use the stronger part of Theorem 2.2.

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